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ORIGINAL PAPER


# M-numerical ranges of odd-order tensors based on operators

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**Abstract**
In this paper, we use tensor norm to define $M$-numerical ranges of odd-order tensors. This notion of numerical range can be useful in the design of fast algorithms for the computation of tensor eigenvalues. Also, we introduce normal tensors based on a product for odd-order tensors. The basic properties of the numerical range of a matrix, such as compactness and convexity, are proved to hold for the $M$-numerical range of an odd-order tensor. We find the $M$-numerical range of a normal tensor. Next, we introduce the singular-value decomposition of an odd-order tensor ($T_M$-SVD), and then use it to find the $M$-numerical range of the tensor.

**Keywords** $M$-numerical range · Odd-order tensor · $T_M$-SVD · $T_M$-EVD · $T_M$-product · Tensor-norm

**Mathematics Subject Classification** 15A69 · 15A60

## 1 Introduction

Let $A \in M_n(\mathbb{C})$ be a $n$-by-$n$ complex matrix. The numerical range of A is defined by

$$W(A) = \{x^*Ax : x \in \mathbb{C}^n, x^*x = 1\}.$$

It is a well-known result due to Toeplitz and Hausdorff that the numerical range is always a convex set. Basic properties and references on the numerical range can be found in [8]. Stampfli and Williams [28, Theorem 4], and later Bonsall and Duncan

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